Chapter 6: Energy

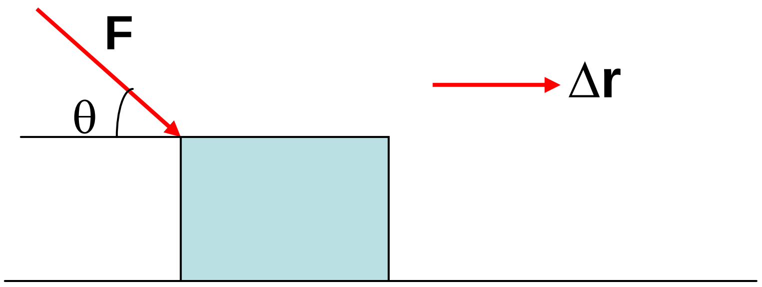
- Work
- Kinetic Energy
- Work by a variable force
- Potential Energy
- Conservation of Mechanical Energy
- Power

§6.2 Work by a Constant Force

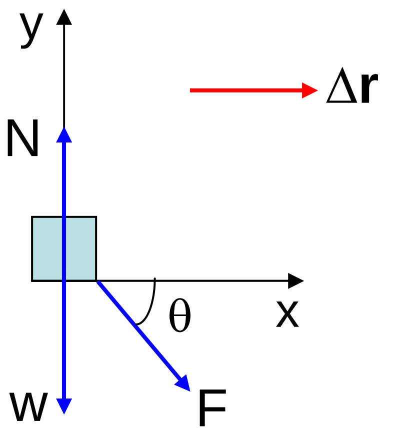
Work is an energy transfer by the application of a force.

The unit of work and energy is the Joule (J). $1\ \text{J} = 1\ \text{Nm} = 1\ \text{kg m}^2/\text{s}^2$.

It is only the force in the direction of the displacement that does work.



An FBD for the crate at left:



The work done by the force **F** is:

$$W_F = F_x \Delta r_x = \left(F\cos\theta\right)\Delta x$$

The work done by the force **N** is: $W_N = 0$

The normal force is perpendicular to the displacement.

The work done by gravity (**w**) is: $W_g = 0$

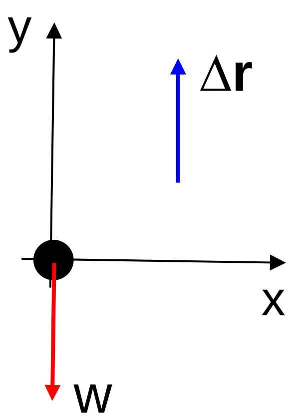
The force of gravity is perpendicular to the displacement.

The net work done on the crate is:

$$\begin{aligned} W_{net} &= W_F + W_N + W_g \\ &= \left(F\cos\theta\right)\Delta x + 0 + 0 \\ &= \left(F\cos\theta\right)\Delta x \end{aligned}$$

Example: A ball is tossed into the air. What is the work by gravity on the way up?

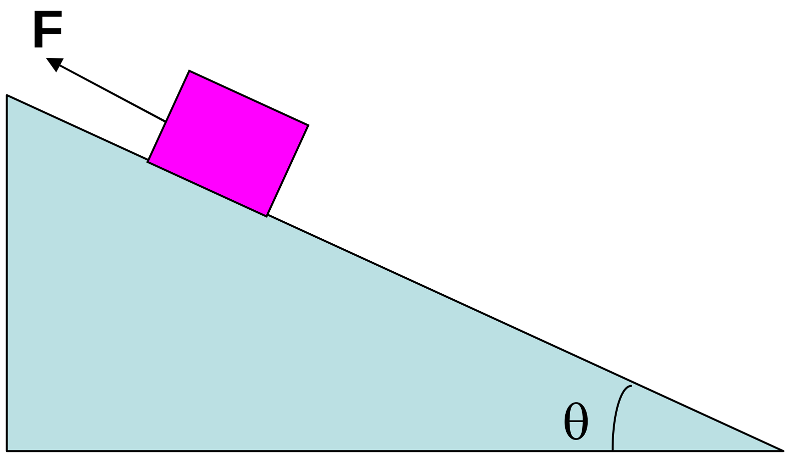
FBD for
rising ball:

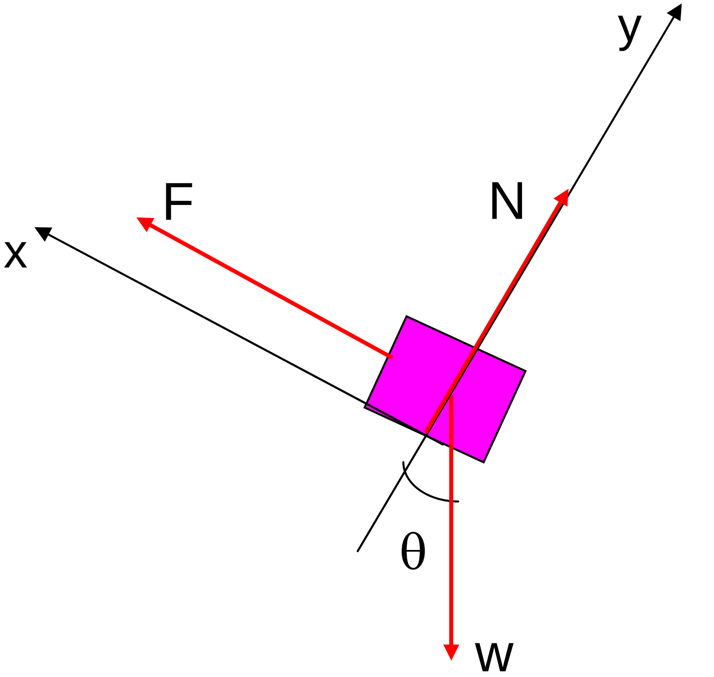


$$W = (-w)(\Delta r_y) = -mg\Delta y$$

Note: the (-) corresponds to energy being removed from the ball.

Example: A box of mass m is towed up a frictionless incline at constant speed. The applied force F is parallel to the incline.





Apply Newton's 2nd law:

$$\sum F_x = F - w\sin\theta = 0$$

$$\sum F_y = N - w\cos\theta = 0$$

What is the work done by the force **F** if the box is *not* pulled at constant speed?

$$\sum F_x = F - w\sin\theta = ma$$

$$\therefore F = ma + w\sin\theta$$

Proceed as before.

§6.3 Kinetic Energy

$$KE = \frac{1}{2}mv^2$$

Is an object's translational kinetic energy.

This is the energy an object has because of its state of motion.

It can be shown that $W_{net} = \Delta KE$

Example (GRR 6.79): The extinction of the dinosaurs and the majority of species on Earth in the Cretaceous Period (65 Myr ago) is thought to have been caused by an asteroid striking the Earth near the Yucatan Peninsula. The resulting ejecta caused widespread global climate change.

If the mass of the asteroid was 10^{16} kg (diameter in the range of 4-9 miles) and had a speed of 30.0 km/sec, what was the asteroid's kinetic energy?

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\left(10^{16}\ \text{kg}\right)\left(30\times 10^3\ \text{m/s}\right)^2$$

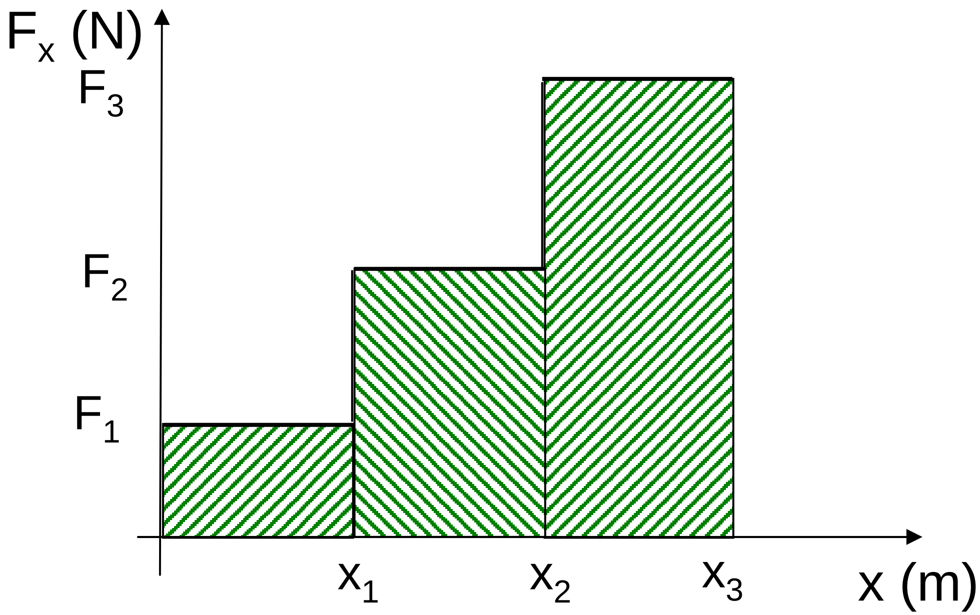
$$= 4.5\times 10^{24}\ \text{J}$$

This is equivalent to ~10^9 Megatons of TNT.

§6.4 Work by a Variable Force

Work can be calculated by finding the area underneath a plot of the applied force in the direction of the displacement versus the displacement.

Example: What is the work done by the variable force F?



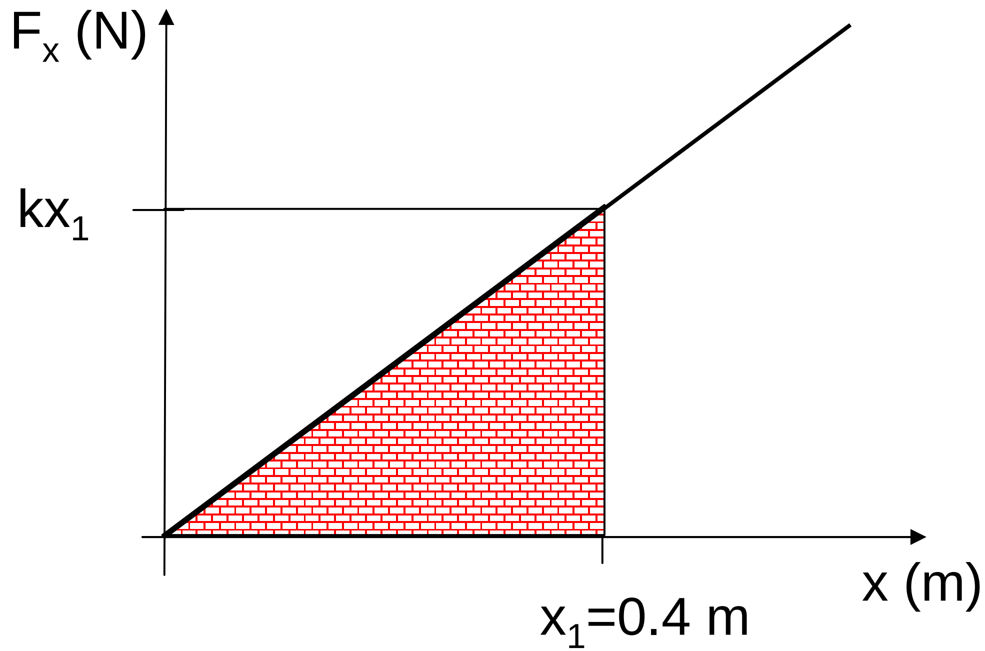
The work done by F_1 is $$W_1 = F_1\left(x_1 - 0\right)$$

The work done by F_2 is $$W_2 = F_2\left(x_2 - x_1\right)$$

The work done by F_3 is $$W_3 = F_3\left(x_3 - x_2\right)$$

The net work is then $W_1+W_2+W_3$.

Example (GRR 6.15) An ideal spring has k=20.0 N/m. What is the amount of work done to stretch the spring 0.40 m from its relaxed length?

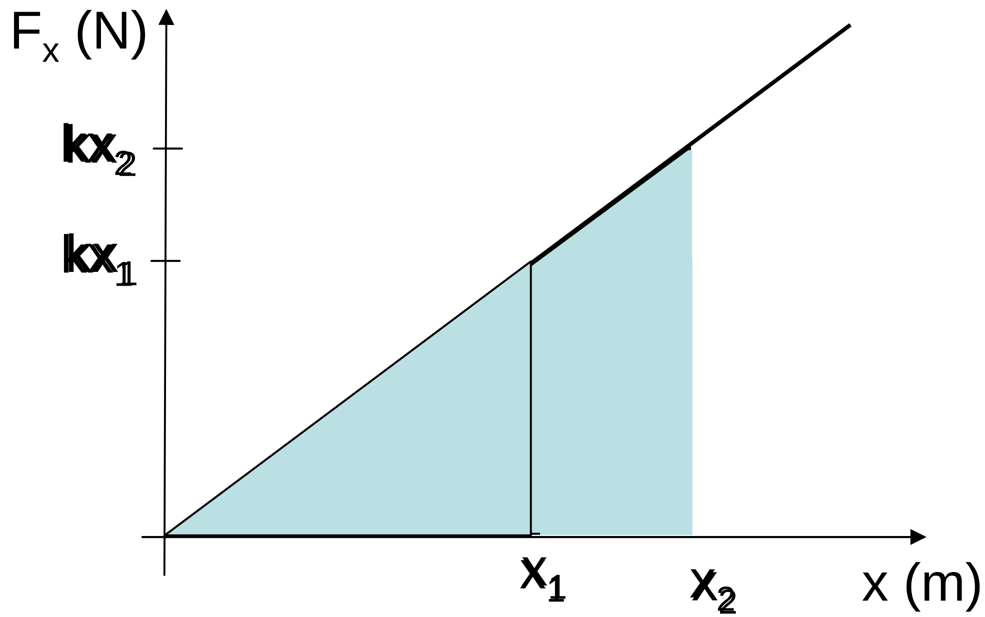


Hooke's Law
F_{spring}=kx.

$$W = \text{Area under curve}$$

$$= \frac{1}{2}(kx_1)(x_1) = \frac{1}{2}kx_1^2 = \frac{1}{2}(20.0\ \text{N/m})(0.4\ \text{m})^2 = 1.6\ \text{J}$$

How much work is done to stretch a spring from x_1 to x_2? (Assume constant speed for the spring.)



$$W = \frac{1}{2}(kx_2)(x_2) - \frac{1}{2}(kx_1)(x_1) = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

§6.5 Potential Energy

Potential energy is the energy an object has because of its location (or configuration).

There are potential energies associated with different (but not all!) forces.

In general $W_{\text{ext}} = \Delta U$

Elastic potential energy-springs:

The work done in stretching/compressing a spring transfers energy to the spring

$$U_s = \frac{1}{2}kx^2$$

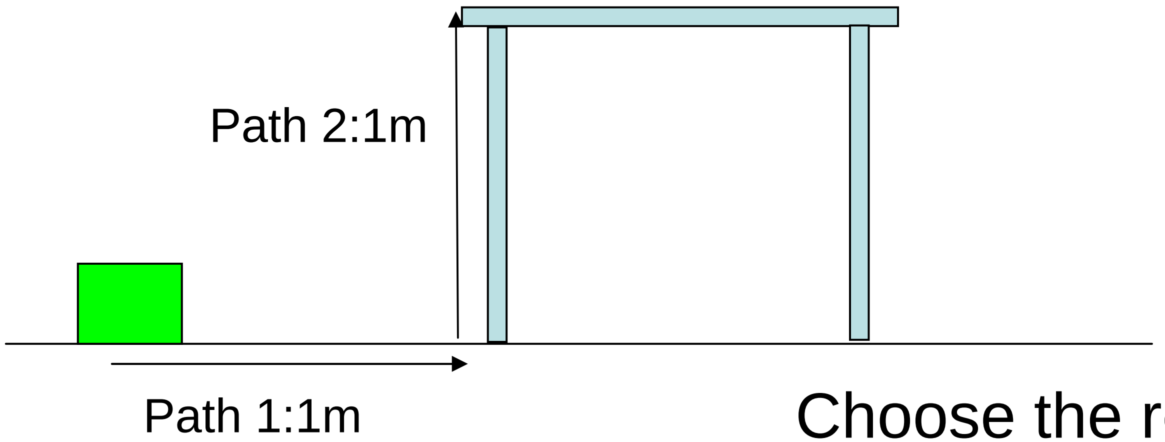
Gravitational potential energy-part 1:

The work done in moving an object in a gravitational field (only near the surface of the Earth) is

$$\Delta U_g = mg\Delta h$$

Δh is the change in the object’s vertical position with respect to some reference point that you are free to chose.

Example: What is the change in the gravitational potential energy of the box if it is placed on the table?



Take m = 1kg

Choose the reference level at the floor. U=0 here.

Along path 1: $\Delta U_g = mg\Delta h = (1\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0\,\mathrm{m}) = 0\,\mathrm{J}$

Along path 2: $\Delta U_g = mg\Delta h = (1\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(1\,\mathrm{m}) = 9.8\,\mathrm{J}$

The value of ΔU is the same for all paths (ending on top of the table). This means $\Delta U = -W_g$ is independent of the path taken (and the reference level used). This is an example of a conservative force.

Name an example of a nonconservative force.

Gravitational potential energy-part 2:

The general expression for gravitational potential energy is

$$U(r) = -\frac{GM_1M_2}{r}$$

$$\text{where}\, U(r = \infty) = 0$$

Example: What is the gravitational potential energy of a body on the surface of the Earth?

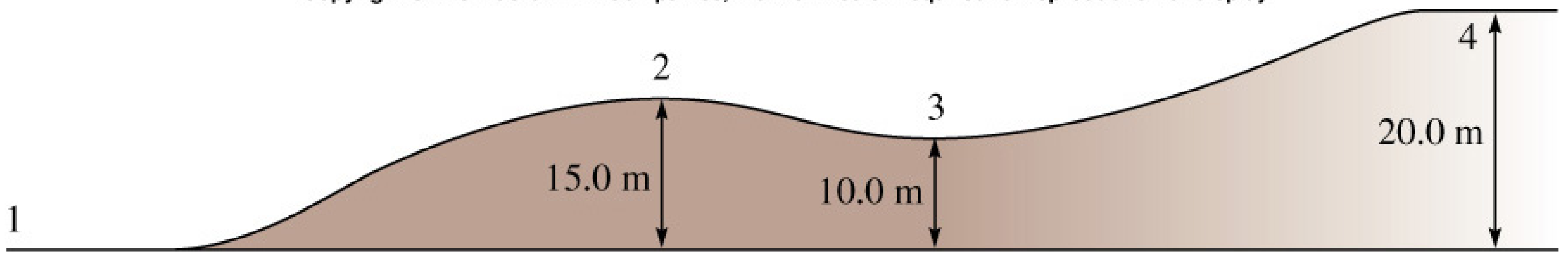
$$U(r = R_e) = -\frac{GM_1M_2}{r} = -\frac{GM_em}{R_e}$$

§6.6 Conservation of Mechanical Energy

The mechanical energy is $$E = K + U$$

Whenever nonconservative forces do no work, then the mechanical energy of a system is conserved. That is $E_i=E_f$ or $\Delta KE= -\Delta U$.

Example (GRR 6.29): A cart starts from position 4 with v = 15.0 m/s to the left. Find the speed of the cart at positions 1, 2, and 3. Ignore friction.



$$E_4 = E_3$$

$$U_4 + KE_4 = U_3 + KE_3$$

$$mgh_4 + \frac{1}{2}mv_4^{\ 2} = mgh_3 + \frac{1}{2}mv_3^{\ 2}$$

$$v_3 = \sqrt{v_4^2 + 2g\left(h_4 - h_3\right)} = 20.5\,\mathrm{m/s}$$

$$E_4 = E_2$$

$$U_4 + KE_4 = U_2 + KE_2$$

$$mgh_4 + \frac{1}{2}m{v_4}^2 = mgh_2 + \frac{1}{2}m{v_2}^2$$

$$v_2 = \sqrt{v_4^2 + 2g\left(h_4 - h_2\right)} = 18.0\ \mathrm{m/s}$$

Or use $E_3 = E_2$

$$E_4 = E_1$$

$$U_4 + KE_4 = U_1 + KE_1$$

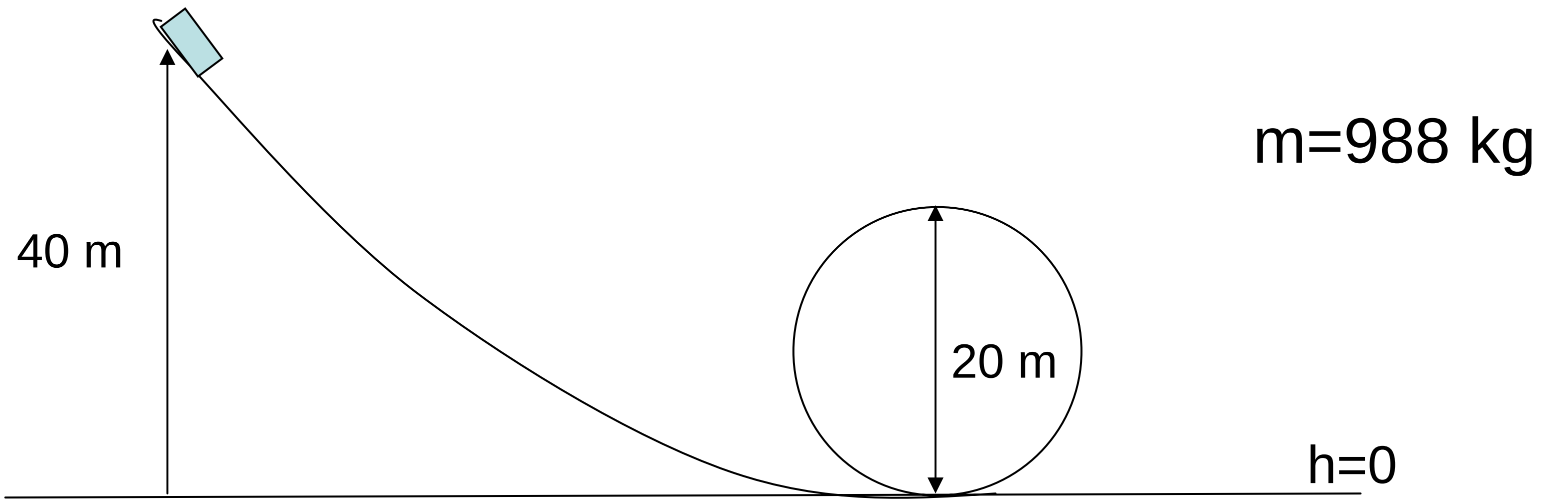
$$mgh_4 + \frac{1}{2}m{v_4}^2 = mgh_1 + \frac{1}{2}m{v_1}^2$$

$$v_2 = \sqrt{v_4^2 + 2g\left(h_4 - h_1\right)} = 24.8\ \mathrm{m/s}$$

Or use $E_3 = E_1$

$E_2 = E_1$

Example (GRR 6.42): A roller coaster car is about to roll down a track. Ignore friction and air resistance.



(a) At what speed does the car reach the top of the loop?

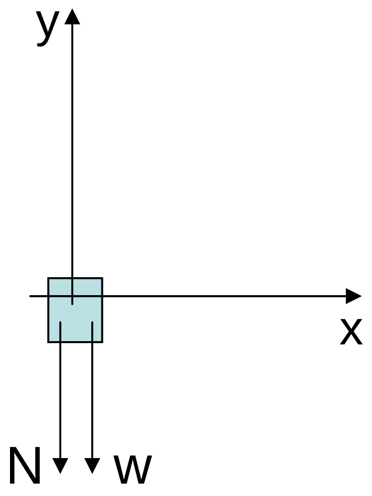
$$E_i = E_f$$

$$U_i + KE_i = U_f + KE_f$$

$$mgh_i + 0 = mgh_f + \frac{1}{2}mv_f^{\ 2}$$

$$v_f = \sqrt{2g\left(h_i - h_f\right)} = 19.8\,\mathrm{m/s}$$

(b) What is the force exerted on the car by the track at the top of the loop?



$$\sum F_y = -N - w = -ma_c = -m\frac{v^2}{r}$$

$$N + w = m\frac{v^2}{r}$$

$$N = m\frac{v^2}{r} - mg = 2.9\times10^4\ \mathrm{N}$$

(c) From what minimum height above the bottom of the track can the car be released so that it does not lose contact with the track at the top of the loop?

At this point N=0 and $v=v_{min}$. This means that

$$N + w = m\frac{v^2}{r}$$

$$w = mg = m\frac{v^2}{r}$$

$$v = \sqrt{gr} = 9.9\,\mathrm{m/s}$$

Using energy conservation:

$$E_i = E_f$$

$$U_i + KE_i = U_f + KE_f$$

$$mgh_i + 0 = mgh_f + \frac{1}{2}mv_f^{\ 2}$$

$$h_i = h_f + \frac{v_f^{\ 2}}{2g} = 25.0\ \text{m}$$

What do you do when nonconservative forces are present? For example, if friction is present

$$E_i = E_f + |W_F|$$

The work done by friction.

§6.7 Power

Average Power $$P_{\text{av}} = \frac{\Delta E}{\Delta t}$$

Instantaneous Power $$P = Fv\cos\theta$$

The units of power: 1 J/s = 1 Watt = 1 W.

Example (GRR 6.56): A race car with a mass of 500.0 kg completes a quarter-mile (402 m) race in a time of 4.2 s. The car's final speed is 125 m/s. What is the engine's power output? Neglect friction and air resistance.

$$P_{\text{av}} = \frac{\Delta E}{\Delta t} = \frac{\Delta U + \Delta KE}{\Delta t}$$

$$= \frac{\Delta KE}{\Delta t} = \frac{\frac{1}{2} m v_f^2}{\Delta t} = 9.3 \times 10^5 \text{ watts}$$

Summary

- Conservation of mechanical energy
- Need to know how to compute work done by a force
- Kinetic energy
- Potential energy (gravitational, elastic)
- Power